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Resonance shift in relativistic traveling wave amplifiers

S. A. Naqvi, G. S. Kerslick, and J. A. Nation
School of Electrical Engineering & Laboratory of Plasma Studies, Cornell University, Ithaca, New York 14853

L. Schächter
Department of Electrical Engineering, Technion–Israel Institute of Technology, Haifa 32000, Israel
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We examine analytically the linear operation of relativistic traveling wave tube amplifiers. In this regime it is found that the maximum growth rate occurs at a beam velocity below that expected on the basis of resonance with the cold dispersion relation of the slow wave structure. The maximum growth rate can be much larger than that at the resonance condition. These results have significance when extending Pierce's theory to traveling wave amplifiers driven by relativistic electron beams.

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In recent years, there has been considerable interest in high power (>100 MW) microwave sources employing relativistic electron beams [1,2]. These devices include klystrons [2], traveling wave amplifiers (TWA's) [3,4], free-electron lasers [5], and various gyro devices [6]. All of these sources operate on the principle of matching the longitudinal or transverse velocity of electrons with the phase velocity of an electromagnetic wave supported by the system. This interaction results in spatial growth of electromagnetic power at the expense of the kinetic energy of the electrons. However, the behavior of these devices in the relativistic regime can differ significantly from their counterparts in the nonrelativistic regime.

In this Brief Report we report results from a study of relativistic traveling wave amplifiers when operated off resonance, i.e., when the dc beam velocity differs from the phase velocity of the electromagnetic wave. It is shown that (i) in the relativistic case, the electron velocity for which maximum spatial growth rate occurs may be substantially lower than the wave phase velocity, i.e., off resonance. This is in contrast to the nonrelativistic case where maximum growth occurs at resonance. (ii) In the relativistic case the maximum growth rate off resonance may be substantially larger than that obtained at resonance. (iii) The interaction bandwidth of a relativistic device may be significantly larger than that of a nonrelativistic device. These characteristics are the result of two competing processes which occur during the interaction. First, the coupling coefficient, which is a measure of the coupling between the electron beam and the wave propagating in the structure, varies as $1/\beta\gamma$ and therefore increases as the electron beam energy is reduced. Second, the slip between the wave and the electrons varies linearly with the beam velocity and causes a decrease in the spatial growth rate as one moves off resonance. In the relativistic case, significant variations may occur in the coupling coefficient prior to any substantial change in the slip since relatively small variations in β may result in large changes in the coupling coefficient. Consequently, the maximum growth occurs at lower velocities than anticipated from the classical resonance condition. In the nonrelativistic regime both parameters depend on β only and the off resonance coupling effects are no longer present.

In this study, we concentrate on the variation of linear growth rate with the beam velocity in a given traveling wave amplifier operated at a single frequency. We use a simplified model in which the slow wave structure consists of a cylindrical waveguide filled with a dielectric through which a beam is launched. The model allows an analytic study of the problem, and while not physically realizable, the model retains the essential physics of the problem.

The axial electric field of a TM_{01} mode in a straight circular waveguide of radius a filled with material of dielectric coefficient ϵ_r is given by

$$E_z(r,z,t)=A_0J_0(k_\perp r)e^{j(\omega-k_0z)}, \tag{1}$$

where $k_\perp=\nu_1/a$ and $\nu_1=2.405$, the first zero of the Bessel function $J_0(x)$. The cold dispersion relation of the structure is given by

$$k_\perp^2=\epsilon_r\frac{\omega^2}{c^2}-k_0^2=\frac{\nu_1^2}{a^2}. \tag{2}$$

The dielectric coefficient ϵ_r is chosen such that electrons of velocity β_0c are synchronous with cold phase velocity ω/k_0.

When the waveguide is filled by a monoenergetic beam of constant density n and dc velocity βc, the dispersion relation for small amplitude waves in the beam waveguide system becomes

$$\left(\epsilon_r\frac{\omega^2}{c^2}-k^2\right)\left(1-\frac{e^2n}{\epsilon_r\epsilon_0(m\gamma)}\frac{1}{[\gamma(\omega-kv)]^2}\right)=\frac{\nu_1^2}{a^2}. \tag{3}$$

The beam current is given by $I=nS\beta ce$, where the beam cross section S is equal to the waveguide cross section πa^2.

Since we shall compare the operation of a relativistic system with nonrelativistic systems, it is convenient to determine the parameters of our simplified model according to the definitions introduced by Pierce [7]. Specifically, we quantify the properties of this slow-wave system in terms of the Pierce gain parameter C^3. At resonance, the normalized spatial growth rate is given by

$$\alpha_0^P = \frac{\text{Im}(k)}{k_0} \approx \frac{\sqrt{3}}{2} C, \tag{4}$$

where

$$C^3 = \left(\frac{1}{2} \frac{Ie}{\gamma mc^2} \frac{\omega}{k_0 \beta c} \frac{1}{\gamma^2 \beta^2} \right) Z_{int} \tag{5}$$

$$\simeq \left(\frac{eI}{2\beta_0^2 \gamma_0^3 mc^2} \right) Z_{int}. \tag{6}$$

The interaction impedance measures the coupling strength of the slow-wave structure and is defined as

$$Z_{int} \equiv \frac{\frac{1}{2}\langle E_z^2 \rangle_S}{k_0^2 P}, \tag{7}$$

where $\langle E_z^2 \rangle_S$ is the square of the axial electric-field averaged over the beam cross section S and P is the average power flow in the waveguide. In the present case it is explicitly given by

$$Z_{int} = \frac{c \eta_0 \nu_1^2}{\pi \epsilon_r a^4 k_0^3 \omega}, \tag{8}$$

where $\eta_0 = 377\Omega$.

In order to make a meaningful comparison between relativistic and nonrelativistic regimes we have to make an adequate choice of parameters in Eq. (3) above. There are four parameters: ω, a, ϵ_r, and the current I. The velocity of the electrons is considered as a variable and the wave number k is the quantity to be determined. Therefore the quantity we shall examine is the normalized gain, defined as

$$\alpha \equiv \frac{\text{Im}(k)}{k_0}. \tag{9}$$

In the discussion which follows, we shall consider a single frequency, 9 GHz, at which we have three slow wave structures designed to be in resonance with beam energies of 1 MeV, 100 keV, and 10 keV, respectively. Consider first the 1 MV case. We shall examine three radii: $a = 20$, 15, and 10 mm. The resonance condition is satisfied by setting

$$\epsilon_r = \frac{1}{\beta_0^2} + \frac{\nu_1^2 c^2}{a^2 \omega^2} \tag{10}$$

for each one of the radii. For adequate comparison with the other two cases (100 kV and 10 kV) we ensure that (i) the normalized growth rate at resonance [calculated from dispersion relation (3)] is kept the same and (ii) the beam term $I/\gamma_0^3\beta_0^2$ in the coupling parameter C is kept constant so that when the first condition is satisfied, Z_{int} is approximately the same for each of the three cases. The second condition determines the current and the former, combined with the requirement for resonance, sets the radius and dielectric constant for the 10 kV and 100 kV cases. There is a unique value of (ϵ_r, a) which satisfies the resonance condition as well as the required value of α at resonance. The scaling results are summarized in Table I.

TABLE I. Parameters at resonance: $(\omega = k_0 \beta_0 c)$.

V_0	Radius a (mm)	I_0 (A)	α_0^P	α_0
1 MV	20	500	0.0295	
100 kV	10.1	11.22	0.0290	0.0275
10 kV	2.65	0.88	0.0285	
1 MV	15	500	0.0407	
100 kV	7.46	11.22	0.0402	0.0385
10 kV	1.89	0.88	0.0395	
1 MV	10	500	0.0613	
100 kV	4.84	11.22	0.0605	0.0586
10 kV	1.15	0.88	0.0600	

Once the design of the slow-wave structure and the beam current are set, the variation of growth rate with beam velocity is calculated using the exact solution of dispersion relation (3). The results are plotted in Fig. 1. We observe that for the relativistic systems, the peak gain occurs when the beam energy is significantly lower than the resonant beam energy. Also, the maximum gain is significantly higher than at resonance. This deviation becomes more pronounced as the coupling strength is increased. To understand this behavior, we see from (5) that the growth rate near resonance varies as $1/\gamma\beta$. As the beam velocity β is decreased below β_0, the coupling strength increases as $\sqrt{1-\beta^2}/\beta$, which in the relativistic case compensates for the increasing slip factor $(\omega - k\beta c)$. For the nonrelativistic case γ is essentially constant and the slight increase in coupling constant with decreasing β is overshadowed by the increasing slip when β deviates from β_0.

An additional insight into this process can be achieved by examining the interaction in the frame of reference in which

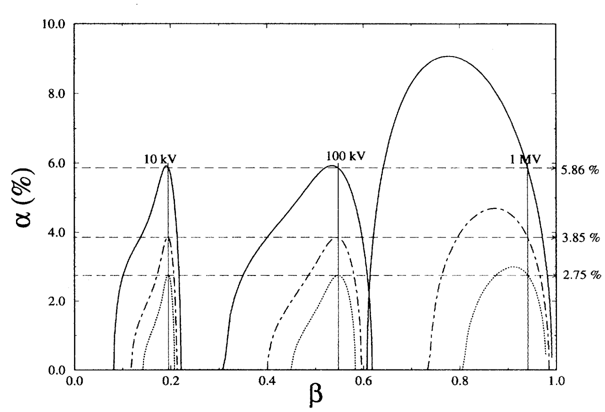

FIG. 1. Normalized growth rate α variation with beam velocity β for 10 kV, 100 kV, and 1 MV systems. The solid curve represents systems which have the value of growth rate at resonance $\alpha_0 = 5.86\%$; the dotted-dashed curve represents $\alpha_0 = 3.85\%$; and the dotted curve represents $\alpha_0 = 2.75\%$. The vertical solid line indicates β_0, the resonant value of β. The dashed horizontal lines indicate α_0, the corresponding value of α at resonance.

the average electron velocity is zero. For this purpose, we use the Lorentz transformation of the (ω,k) 4-vector into the frame moving with velocity βc. The transformed vector (denoted by a prime) reads

$$\begin{pmatrix} \omega' \\ k' \end{pmatrix} = \begin{pmatrix} \gamma(\omega-\beta ck) \\ \gamma(k-\beta\omega/c) \end{pmatrix} \tag{11}$$

of which the growing components read

$$\mathrm{Im}\begin{pmatrix} \omega' \\ k' \end{pmatrix} = \begin{pmatrix} -\beta\gamma ck_0\alpha \\ \gamma\alpha k_0 \end{pmatrix}. \tag{12}$$

Since we examined the spatial growth rate in the laboratory frame in Fig. 1, it is natural now to ask how this quantity varies when measured in the moving frame. We therefore replot the data from Fig. 1 by graphing in Fig. 2 the product $\gamma\alpha$ as a function of β. It can be observed that the curves have a similar shape for both relativistic and nonrelativistic regimes and peaks very close to the classical resonance $\beta=\omega/k_0$. For example, in the 1 MV system described above ($\alpha_0=5.86\%$), α peaks at 300 kV, whereas $\gamma\alpha$ peaks at 1.00 MV, i.e., at resonance. However, it should be noted that it is α that determines the amount of beam power converted to radiation and therefore this relativistic shift needs to be considered in the design of traveling wave amplifiers operating in this regime. Thus when extending Pierce theory to traveling wave tubes driven by relativistic beams it is essential to remember that it is the quantity $\gamma\alpha$ rather than α that peaks at resonance.

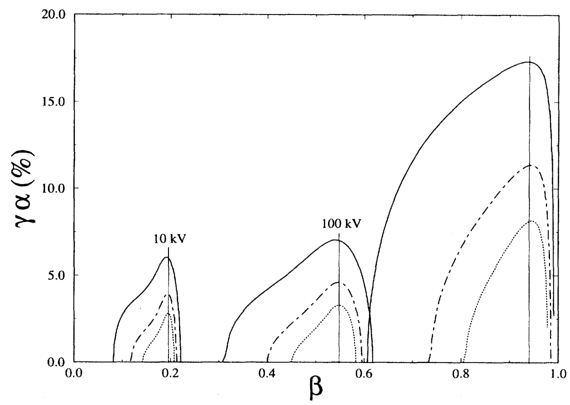

FIG. 2. $\gamma\alpha$ as a function of β for 10 kV, 100 kV, and 1 MV systems. The solid curve represents systems which have the value of growth rate at resonance $\alpha_0=5.86\%$; the dotted-dashed curve represents $\alpha_0=3.85\%$; and the dotted curve represents $\alpha_0=2.75\%$. The vertical solid line indicates β_0, the resonant value of β. The dashed horizontal lines indicate α_0, the corresponding value of α at resonance.

In this Brief Report, we examined the linear operation of relativistic and nonrelativistic traveling wave amplifiers. We observed that (i) in the relativistic regime the maximum growth rate does not occur at resonance but at a beam velocity lower than the phase velocity of the cold structure, (ii) this maximum growth rate can be much larger than the growth rate at resonance, (iii) for both regimes it is the product $\gamma\alpha$ that peaks at resonance , and (iv) relativistic devices are expected to be less sensitive to beam voltage fluctuations than their nonrelativistic counterparts. This relative insensitivity is expected to be significant, for example, in the design of modulators used to produce the electron beams which drive high power traveling wave amplifiers.

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